crements is an element of quantity. Increments and differentials are not identical. The former are vested with quantity, while the latter are vested with quality, i. e., they are formal in their nature.

Milwaukee, Wisconsin, September, 1896.

A PROBLEM IN ASTRONOMY.

By G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

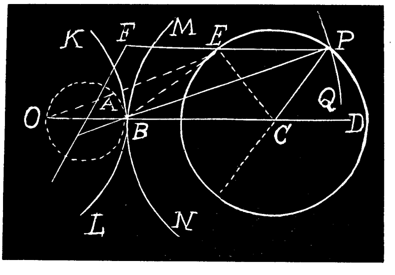
To find the Distance from the Earth to the Sun knowing the distance from an Inferior planet to the Sun supposing the planets to describe circles around the Sun.

Let P be a point on the epicyclic curve PQ, OC the radius of the deferent, CP the radius of the epicycle. Let $CO : CB = n : 1$. $\therefore CB = \frac{CO}{n}$.

Then $BO = CO - CB = CO - \frac{CO}{n} = CO(1 - \frac{1}{n})$.

Now the angular velocity$=\frac{\text{transverse velocity}}{\text{radius vector}}$.

The transverse velocity of E is represented by EA in magnitude, and in direction by BA. Let the linear velocity of the mean point (C) be V, the linear velocity of the moving point in the epicycle is



$$nV.\frac{PC}{CO}.$$

$\therefore$ The transverse velocity$=$

$$\frac{nV}{CO}.EB\cos BEO;$$

$$\text{but } \cos BEO = \frac{(BE)^2 + (EO)^2 - (BO)^2}{2BE.EO}.$$

$\therefore$ Transverse velocity $=\frac{nV}{CO}.\frac{BE^2 + EO^2 - BO^2}{2EO}$. Also radius vector$=EO$.

$$\therefore \text{Angular velocity} = \frac{nV}{CO}.\frac{BE^2 + EO^2 - BO^2}{2EO^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (A).$$

Let the deferential angle$=\theta$, then angle $ECD=(n-1)\theta$.

$\therefore\ BE^2=CE^2+CB^2+2CE.CB\cos(n-1)\theta$

$$=CE^2+\frac{CO^2}{n^2}+2CE.\frac{CO}{n}\cos(n-1)\theta$$

$$EO^2=CO^2+CE^2+2CO.CE\cos(n-1)\theta$$

$$BO^2=CO^2\left(1-\frac{1}{n}\right)^2.$$

Substituting these values in (A) we get

$$\text{Angular velocity}=\frac{V}{CO}.\frac{CO^2+nCE^2+(n+1)CO.CE\cos(n-1)\theta}{CO^2+CE^2+2CO.CE\cos(n-1)\theta}.$$

Now in inferior conjunction, if the moving planet is inferior, $(n-1)\theta=180°$.

$$\therefore\ \text{Angular velocity}=\frac{V}{CO}.\frac{CO^2+nCE^2-(n+1)CO.CE}{CO^2+CE^2-2CO.CE}.$$

Let $CO=R,\ CE=r$.

$$\text{Then Angular velocity}=\frac{V}{R}.\frac{R^2+nr^2-(n+1)R.r}{(R-r)^2}.$$

Now $n=\left(\frac{R}{r}\right)^{\frac{3}{2}}$, also putting $\frac{V}{R}=\omega$.

$$\therefore\ \text{Angular velocity}=\omega.\frac{R^2+\left(\frac{R}{r}\right)^{\frac{3}{2}}r^2-\left\{\left(\frac{R}{r}\right)^{\frac{3}{2}}+1\right\}R.r}{(R-r)^2}$$

$$=\omega\frac{1+\left(\frac{R}{r}\right)^{\frac{3}{2}}\frac{r^2}{R^2}-\frac{R^{\frac{3}{2}}+r^{\frac{3}{2}}}{r^{\frac{3}{2}}}.\frac{r}{R}}{\left(1-\frac{r}{R}\right)^2}\ \ldots\ldots\ldots\ldots\ldots(B)$$

$$=\omega\frac{\frac{R^2}{r^2}+\left(\frac{R}{r}\right)^{\frac{3}{2}}-\left\{\left(\frac{R}{r}\right)^{\frac{3}{2}}+1\right\}\frac{R}{r}}{\left(\frac{R}{r}-1\right)^2}\ \ldots\ldots\ldots\ldots(C).$$

Let the distance from the earth to the sun be known to find the distance from the planet to the sun.

Let $\frac{r}{R}=\rho$, then (B) becomes

$$\text{Angular velocity}=\omega.\frac{1+\rho^{\frac{1}{2}}-\rho^{-\frac{1}{2}}-\rho}{(1-\rho)^2}=\omega\,.\,\frac{(1-\rho)-\rho^{-\frac{1}{2}}(1-\rho)}{(1-\rho)^2}$$

$$=\omega.\frac{1-\rho^{-\frac{1}{2}}}{1-\rho}=-\frac{\omega}{\sqrt{\rho}}.\frac{1-\rho^{\frac{1}{2}}}{1-\rho}=-\frac{\omega}{\sqrt{\rho}+\rho}\;\ldots\ldots\ldots\ldots(D).$$

Let the distance from the planet to the sun be known to find the distance from the earth to the sun.

Let $\frac{R}{r}=\rho'$, then (C) becomes

$$\text{Angular velocity}=\omega.\frac{\rho'^2+\rho'^{\frac{3}{2}}-\rho'^{\frac{5}{2}}-\rho'}{(\rho'-1)^2}=\omega\frac{\rho'(\rho'-1)-\rho'^{\frac{3}{2}}(\rho'-1)}{(\rho'-1)^2}$$

$$=\omega.\frac{\rho'-\rho'^{\frac{3}{2}}}{\rho'-1}=-\frac{\omega\rho'}{1+\sqrt{\rho'}}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(E).$$

Case I. A planet transits the sun's disc at such a rate that the sun's diameter S would be traversed in time t. Find the planet's distance from the sun.

Let ρ=planet's distance, unity being the earth's distance, and let ω be the earth's angular velocity around the sun=sun's angular velocity around the earth, and let t' be the time in which the sun in his annual course moves through a distance equal to his own apparent diameter; then $\omega t'=S$. From (D) the planet's angular velocity about the earth$=-\frac{\omega}{\sqrt{\rho}+\rho}$.

∴. That is the planet's retrograde gain on the sun is

$$\frac{\omega}{\sqrt{\rho}+\rho}+\omega=\frac{S}{t}=\frac{\omega t'}{t}.$$

$$\therefore\ \rho+\rho^{\frac{1}{2}}=\frac{t}{t'-t},\quad \therefore\ \rho^{\frac{1}{2}}=\tfrac{1}{2}\left(\pm\frac{\sqrt{3t+t'}}{\sqrt{t'-t}}-1\right).$$

$$\therefore\ \rho=\tfrac{1}{2}\left(\frac{t'+t}{t'-t}-\sqrt{\frac{3t+t'}{t'-t}}\right)\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Case II. If we wish to find the earth's distance knowing the planet's distance, then let the planet's distance be unity and the earth's distance$=\rho'$.

Proceeding the same as before using (E) we get

$$\frac{\omega\rho'}{1+\sqrt{\rho'}}+\omega=\frac{S}{t}=\frac{\omega t'}{t}.\quad \therefore\ \rho'-\frac{t'-t}{t}\sqrt{\rho'}=\frac{t'-t}{t};$$

$$\therefore\ \sqrt{\rho'}=\frac{1}{2t}\left\{(t'-t)+\sqrt{t'^2-3t^2+2tt'}\right\}.$$

$$\therefore\ \rho'=\frac{t'-t}{2t^2}\left\{(t'+t)+\sqrt{t'^2-3t^2+2tt'}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Suppose Venus transits the sun's disc at such a rate that the sun's apparent diameter would be traversed in 7⅓ hours, and at the same time the sun in his annual course moves through a distance equal to his own apparent diameter in 12 hours. Required (1) the distance from Venus to the sun, the earth's distance being unity, and (2) the distance from the earth to the sun, Venus's distance being unity.

Now $t=7\frac{1}{3}$, $t'=12$; hence for first case substituting in (1)

$$\rho=\tfrac{1}{2}\left(\tfrac{29}{7}-\sqrt{\tfrac{51}{7}}\right)=.721824.$$

For the second case substitute in (2)

$$\rho'=\tfrac{7}{484}\{58+\sqrt{1428}\}=1.38538.$$

(The above is suggested in Proctor's Geometry of the Cycloid.)

A PROPOSITION IN DETERMINANTS.

By ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

THEOREM.—The product of two numbers, each the sum of four squares, is the sum of eight squares.

$$\begin{vmatrix} a+b\sqrt{-1} & -c+d\sqrt{-1} \\ c+d\sqrt{-1} & a-b\sqrt{-1} \end{vmatrix} \times \begin{vmatrix} \alpha+\beta\sqrt{-1} & -\gamma+\delta\sqrt{-1} \\ \gamma+\delta\sqrt{-1} & \alpha-\beta\sqrt{-1} \end{vmatrix}$$

$$=\begin{vmatrix} a+b\sqrt{-1} & -c+d\sqrt{-1} & 0 \\ c+d\sqrt{-1} & a-b\sqrt{-1} & 0 \\ 0 & 0 & 1 \end{vmatrix} \times(-1) \begin{vmatrix} \alpha+\beta\sqrt{-1} & 0 & -\gamma+\delta\sqrt{-1} \\ \gamma+\delta\sqrt{-1} & 0 & \alpha-\beta\sqrt{-1} \\ 0 & 1 & 0 \end{vmatrix}$$

$$=(-1)\begin{vmatrix} a\alpha-b\beta+(a\beta+b\alpha)\sqrt{-1} & a\gamma-b\delta+(a\delta+b\gamma)\sqrt{-1} & -c+d\sqrt{-1} \\ c\alpha-d\beta+(c\beta+d\alpha)\sqrt{-1} & c\gamma-d\delta+(c\delta+d\gamma)\sqrt{-1} & a-b\sqrt{-1} \\ -\gamma+\delta\sqrt{-1} & \alpha-\beta\sqrt{-1} & 0 \end{vmatrix}$$